Micro C - Spring 2015 - Exam Solutions

1. Consider the following game F, where Player 1 chooses the row and Player 2 simultaneously chooses the column.

		Player 2		
		D	E	F
	A	$3,4$	$3,1$	$3,3$
Player 1	B	$2,3$	$1,4$	$2,2$
	C	$3,3$	$1,0$	$5,5$

   (a) Briefly explain whether F is a game of complete or of incomplete information (1 sentence).

   SOLUTION: By definition, F is a game of complete information, since each player knows the payoff function of the other.

   (b) Find all Nash equilibria (pure and mixed) in F.

   SOLUTION: The pure strategy Nash equilibria are (A, D) and (C, F). They can be found by underlining the highest payoff for Player 1 in each row and for Player 2 in each column. Notice that B is strictly dominated by A, and after B is removed from the game, E is strictly dominated by D. Hence, in a mixed strategy Nash equilibrium, Player 1 must be indifferent between A and C, and/or Player 2 must be indifferent between D and F. The set of MSNE is given by $(p \in [2/3, 1), q = 1)$: Player 2 chooses D with probability 1, whilst Player 1 chooses A with probability $p \geq 2/3$ and C with probability $1 - p$.

   (c) Look back at your answer to part (b). If you found a unique Nash equilibrium, explain the intuition as to why this equilibrium is unique. If you found multiple Nash equilibria, argue which of these equilibria is most reasonable. (2-3 sentences)

   SOLUTION: The Nash equilibrium (C, F) might seem most reasonable because it is Pareto dominant, and also because any other Nash equilibrium has Player 1 place strictly positive probability on a strategy that is weakly dominated (after iterated elimination).

2. Now consider the game $G(2)$, with stage game G given by:

		Player 2		
		D	E	F
	A	$6,8$	$0,9$	$1,3$
Player 1	B	$2,0$	$3,3$	$2,2$
	C	$6,2$	$2,1$	$5,4$

   (a) Find the Subgame Perfect Nash Equilibrium (SPNE) of $G(2)$ that gives the higher total payoff to both players. Find the SPNE of $G(2)$ that gives the lowest total payoff to both players. Argue briefly whether or not each SPNE is also a Nash equilibrium (1-2 sentences).

SOLUTION: The two pure strategy Nash equilibria of G are (B, E) and (C, F). Hence, there is a SPNE of $G(2)$ where Player 1 uses strategy "Always play B" and where Player 2 uses strategy "Always play E", giving a total payoff of 6 to each. This is the SPNE that gives the lowest total payoff to both players (notice that the stage-game payoffs from (C, F) are larger than those from (B, E)). There is another SPNE which is as follows: Player 1 plays A in period 1. He plays C in period 2 if the outcome in period 1 was (A, D), and otherwise he plays B. Player 2 plays D in period 1. He plays F in period 2 if the outcome in period 1 was (A, D), and otherwise he plays E. Neither player has an incentive to deviate in period 2, since (B, E) and (C, F) are both Nash Equilibria of G. Taking into account payoffs in period 2, the strategic situation in period 1 can be represented by the following game (reduced form):

		Player 2		
		D	E	F
	A	$11, 12$	$3, 12$	$4, 6$
Player 1	B	$5, 3$	$6, 6$	$5, 5$
	C	$9, 5$	$5, 4$	$8, 7$

(A, D) is a Nash Equilibrium of this game, which means that the original strategy profile for $G(2)$ is subgame perfect. This SPNE gives the highest total payoff to both players (notice that they each earn a higher per-period payoff than in any Nash equilibrium of G). By definition all SPNE are also Nash equilibria, because strategy profiles must constitute a Nash equilibrium in every subgame.

(b) Suppose that the payoff $(3, 3)$ from strategy profile (B, E) is replaced by $(3, 4)$. How does this change affect the two SPNE you found in part (a)? Discuss whether you would expect Player 2 to benefit from this change (2-3 sentences).

SOLUTION: The pure strategy Nash equilibria of the stage game remain (B, E) and (C, F), so there is still a 'low-payoff' SPNE of $G(2)$ where players always play (B, E) in both periods. But the 'high-payoff' SPNE described in part (a) no longer exists, since Player 2 can profitably deviate to E in period 1 and earn a total payoff of 13. Player 2 benefits from the change in payoffs if the players were originally going to play the low-payoff SPNE of $G(2)$ but not if they were going to play the high-payoff SPNE.

(c) Suppose that the payoff $(3, 3)$ from strategy profile (B, E) is replaced by $(4, 3)$. How does this change affect the two SPNE you found in part (a)? Discuss briefly whether you expect Player 1 to benefit from this change (2-3 sentences). Explain how your answer compares to that in part (b) (2-3 sentences).

SOLUTION: In this case, both the low-payoff SPNE and the high-payoff SPNE of $G(2)$ continue to exist. This suggests that Player 1 should at least weakly benefit from this change in payoffs. The difference from part (b) is that Player 1 is not tempted to deviate in period 1 if he plays A and player 2 plays D, so increasing his payoff in the 'punishment' outcome (B, E) has no impact on his period-1 incentives. In contrast, Player 2 will only play D in period 1 if he fears punishment in period 2, so increasing his payoff from the possible punishment in period 2 can lead him to deviate from D in period 1.

3. Two professors each want the same student to get a job at the University of Copenhagen. The student is sure to get the job as long as she receives a glowing and personal reference

from at least one of the professors. But writing such a reference take time: the time-cost of professor i is c_i, which is private information. The two professors must simultaneously decide whether or not to write the reference, with their payoffs given by:

		Prof. 2 Write	Prof. 2 Don't
Prof. 1	Write	$1-c_1, 1-c_2$	$1-c_1, 1$
	Don't	$1, 1-c_2$	$0, 0$

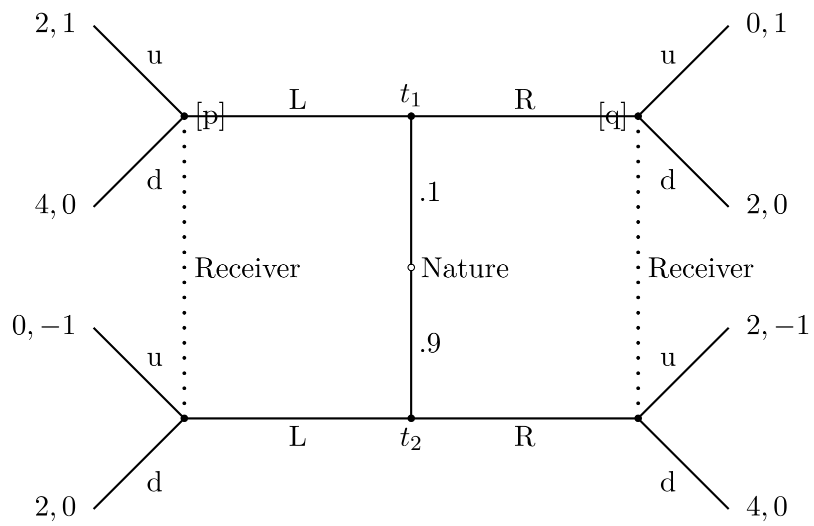
(a) Suppose that $c_1 = c_2 = 1/4$ with probability 1. Briefly describe the main similarity and the main difference between the game played by the Professors and (i) the Prisoners' Dilemma, (ii) the Stag Hunt game, and (iii) the Chicken game (1-2 sentences for each of (i)-(iii)). Who will end up writing the reference?

Possible answers include the following: (i) similarity: professors' interests are not fully aligned, difference: no professor has a dominant strategy; (ii) similarity: professors both benefit from coordinating their actions, difference: no outcome is Pareto dominant; (iii) similarity: professors both benefit from coordinating their actions, interests are not fully aligned, difference: difficult to say, as the professors are effectively playing a game of chicken. There are two pure strategy Nash equilibria, each where a different professor writes the reference. The game is entirely symmetric so there is no particular reason to think one of these equilibria is more likely than the other.

(b) Continue to assume that $c_1 = 1/4$ with probability 1. But now suppose that $c_2 = 1/4$ with probability θ and $c_2 = 2$ with probability $1-\theta$, where $\theta < 2/3$. Find the Bayesian Nash equilibrium of this game. Who will end up writing the reference? Describe briefly how your answer compares to that in part (a), and why this is the case (2-3 sentences).

The high-cost type of Professor 2 has a strictly dominant strategy not to write the reference. This means that Professor 1 finds it optimal to write the reference, regardless of the action of the low-cost type of Professor 2: $3/4 > 2/3 > \theta = \theta(1) + (1-\theta)0$. *Hence, Professor 1 plays "Write", and the best reply of Professor 2 is to play "Don't" regardless of type. The difference with part (a) is that now Professor 1 finds it sufficiently likely that Professor 2 would never write the letter, that it is worthwhile for Professor 1 to write the letter himself (which in particular benefits Professor 2).*

4. Now consider the following game G':



(a) Briefly explain whether G' is a static or a dynamic game (1 sentence).

SOLUTION: By definition, G is a dynamic game, where the Sender chooses a message and the Receiver then responds with an action.

(b) Find a separating equilibrium in G', and find a pooling equilibrium where both sender types play L.

SOLUTION: Separating equilibrium: $(LR, ud, p = 1, q = 0)$. *Pooling equilibria:* $(LL, du, p = 0.1, q \geq 1/2)$.

(c) Check whether the equilibria you found in part (b) satisfy Signaling Requirements 5 and 6.

SOLUTION: The separating equilibrium satisfies both signaling requirements, since there are no off-the-equilibrium-path beliefs, and hence nothing to check. The pooling equilibrium satisfies Signaling Requirement 5, since no sender type has a strictly dominated strategy. The pooling equilibrium does not satisfy Signaling Requirement 6; t_1*'s equilibrium payoff of* 4 *is strictly higher than any payoff he could possibly get by deviating to* R*, whereas* t_2*'s equilibrium payoff of* 2 *is not. Signaling Requirement 6 therefore implies* $q = 0$*, whereas the pooling equilibrium required* $q \geq 1/2$.

(d) Describe a hypothetical real-world strategic situation that could correspond to G', and explain why this is the case (3-4 sentences). What (if anything) do your answers to parts (b) and (c) suggest about the behavior we are likely to see in this real-world situation? (2-3 sentences)

SOLUTION: One possible answer is as follows. The Sender could be a CEO with two possible levels of skill in dealing with competition (t_1 *is low-skill,* t_2 *is high-skill). Each message could be a possible educational degree for the CEO, where the low-skill (high-skill) CEO finds education R (L) more costly. The Receiver could be another firm deciding whether to compete (u) or not (d) with the CEO's firm in the product market. The analysis above might suggest that the CEO takes an education level that reveals his skill, and the other firm decides to compete if and only if this skill level is revealed to be low (separating equilibrium).*